PARCC Model Content Frameworks

A Companion to the Common Core State Standards

Mathematics:

Kindergarten through Grade 2

September 2014

PARCC
Partnership for Assessment of
Readiness for College and Careers

Overview of the Kindergarten through Grade 2 Model Content Frameworks

The PARCC K-2 Model Content Frameworks are voluntary resources meant to be used as a companion to the Common Core State Standards to help educators and those developing aligned curricula and instructional materials. The frameworks help clarify the standards by illustrating how key content shifts from Kindergarten through Grade 2 coherently to Grade 3 and beyond. They also serve as an example of how teachers and curriculum writers may frame instruction using the standards across the academic year. The Model Content Frameworks are neither a curriculum nor a replacement to the standards. Rather, they ought to be used as a companion to the standards, and as a lens through which to analyze and build local curricula.

The Model Content Frameworks for Kindergarten through Grade 2—one for mathematics and one for English language arts — were developed by PARCC state representatives, educators, and experts in academic standards and early learning, instruction, and formative assessment. Public feedback from teachers across the United States helped to shape the final versions of the frameworks published here. Unlike their later-grade counterparts, the Model Content Frameworks for K-2 are not focused on connections to summative assessments. Rather, they are designed to support the development of classroom-level, non-summative tools like PARCC's formative tasks and diagnostic assessments.

PARCC has released several prototype K-2 formative tasks which demonstrate concepts presented in these frameworks. PARCC plans to release new K-2 formative tasks to exemplify the standards, aligned to these Model Content Frameworks for K-2 in August, 2015.

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INTRODUCTION TO THE PARCC MODEL CONTENT FRAMEWORKS FOR MATHEMATICS: KINDERGARTEN THROUGH GRADE 2

Purpose of the Kindergarten through Grade 2 Model Content Frameworks for Mathematics

As part of its proposal to the U.S. Department of Education, the Partnership for Assessment of Readiness for College and Careers (PARCC) committed to developing Kindergarten through Grade 2 Model Content Frameworks for Mathematics (K–2 Model Content Frameworks) to serve as a bridge between the Common Core State Standards (CCSS) and the PARCC non-summative tools. PARCC developed the K–2 Model Content Frameworks to help:

- guide the development of diagnostic tools and formative instructional tasks that align to the CCSS and vertically align to the PARCC Assessment System;
- support the implementation of the CCSS; and
- address areas of emphasis that develop college and career readiness in early grades.

The PARCC K–2 Model Content Frameworks were developed through a state-led process that included mathematics content experts in PARCC member states and members of the CCSS writing team. Although the primary purpose of the K–2 Model Content Frameworks is to provide a frame for the PARCC K–2 diagnostic and formative tasks, they are also voluntary resources to help educators and those developing curricula and instructional materials. Users are advised to have a copy of the CCSS available for use in conjunction with the K–2 Model Content Frameworks.

Connections to the PARCC K–2 Formative Tasks and Grade 2 Diagnostic Tools

The PARCC K–2 formative instructional tasks and grade 2 diagnostic tools will be designed to gather evidence, for real-time instructional uses, of the knowledge, skills, and understandings that students have at a given moment and that are essential for laying foundations of college and career readiness in the early years. In mathematics, these include age-appropriate conceptual understanding, procedural skill and fluency, and application and problem solving, as defined by the standards. In early childhood education, these also include non-high-stakes testing environments that nurture meaningful mathematical experiences, and in which knowledgeable educator observations of children engaging in meaningful tasks inform and guide instruction toward focus and coherence.

Each of these components works in conjunction with the others to promote students' achievement in mathematics as early learners gain trust in the world around them to construct knowledge by authentically developing mathematical practices that support understandings of mathematical content. To measure the levels of student understanding of the standards, the K–2 formative instructional tasks and grade 2 diagnostic tools will include various tasks that provide real-time qualitative and/or quantitative data on how children are or are not making progress toward the standards and how to provide sufficient practice and extra support for all children to meet the standards.

For decades, researchers have almost universally concluded that high-quality early childhood education is a key factor for academic success in later grades.[1] Very early work with numbers is rich and intricate, and requires time and repeated experiences for young learners. The K–2 Model Content Frameworks for Mathematics reflect these priorities by providing detailed information about selected practice standards, fluencies, connections, and content emphases. These emphases will be reflected in the diagnostic tools and formative instructional tasks to lay the groundwork for success with the PARCC Assessment System in later grades.

The K–2 Model Content Frameworks do not contain a suggested scope and sequence for a specific period of time. Rather, they provide examples of key content dependencies (where one concept ought to come before another), key instructional emphases, opportunities for in-depth work on key concepts, and connections to critical practices. These last two components, in particular, intend to support local and state efforts to deliver instruction that connects content and practices while achieving the standards' balance of conceptual understanding, procedural skill and fluency, and application.

Overall, the K–2 PARCC diagnostic tools and formative instructional tasks will include teacher-led activities, observations, and questions that elicit evidence informing instruction about whether students can:

- solve problems involving the major clusters of the grade, with connections to the practice standards;
- express mathematical reasoning by constructing mathematical arguments and critiques;
- solve real-world problems by engaging, particularly in the modeling practice; and
- demonstrate fluency in the areas set forth in the content standards for grades K–2.

Diagnostic tools and formative instructional tasks will measure various levels of understanding as students demonstrate and apply mathematical reasoning with discrete objects, pictorial representations, or symbolic expressions and equations required within and across mathematical domains and practices. These student demonstrations will provide teachers with the qualitative and/or quantitative data needed to ensure that all students receive the necessary experiences and support to achieve the standards. They will address an age-appropriate range of mathematics, including conceptual understanding, procedural fluency, and the varieties of practice standards developing among early learners.

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[1] National Research Council. *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity* (Washington, DC: The National Academies Press, 2009).

The Common Core and Early Childhood Education

States have had standards for K–2 mathematics for a long time—since well before the development of the CCSS. The CCSS build on states' long-standing work in the primary grades and are designed to provide young students with a rich, rewarding, rigorous, and equitable education.

Experts in early childhood mathematics education were directly involved in developing the CCSS for Mathematics. The standards were also informed by a range of evidence, including peer-reviewed empirical studies, summaries of research in education, recommendations from mathematicians and mathematics educators, expectations from previous state standards documents, expectations from high-performing countries, and domestic reports such as *Adding It Up*[2] and *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*[3]. Drafts of the standards were reviewed by experts in early childhood education as well as by current and former elementary-grades teachers. Between them, the working group and the feedback group included (among others) the following:

- state directors of P–16 and P–20 programs, responsible for public pre-K and early elementary education in their states;
- current and former public-school elementary-grades teachers;
- Deborah Ball, an eminent researcher on mathematics education, who also taught elementary school for 15 years; and
- Karen Fuson and Douglas Clements, eminent mathematics education researchers who work in school settings alongside young children. Clements is a Kennedy Endowed Chair in Early Childhood Learning and the Executive Director of the Marsico Institute of Early Learning and Literacy; Fuson and Clements were also members of the National Research Council's Early Math Panel.

Input from experienced K–2 teachers was also solicited and used. For example, the connections between numbers and the quantities they name were improved based on the input of K–2 teachers who were part of a larger group convened by the American Federation of Teachers for the purpose of providing feedback on an early draft of the standards. Experts in early childhood education from other organizations also provided feedback on the public draft. This led to concrete changes in the final version. For example, the concept of a unit of one ten was moved from kindergarten to grade 1 in response to feedback from early childhood educators. Research on early learning strongly informed the early-grades standards. For example, the National Research Council's 2009 report *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity* is cited in the bibliography of the CCSS (pp. 88–89).

In 2010, the National Association for the Education of Young Children and the National Association of Early Childhood Specialists in State Departments of Education issued a joint statement about the public draft of the CCSS, including some important cautions and good issues to think about; their bottom-line verdict on the draft was that it is ***"fair and age appropriate for kindergarten through 3rd grade."***

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[2] National Research Council. *Adding It Up: Helping Children Learn Mathematics* (Washington, DC: The National Academies Press, 2001).

[3] National Research Council. *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity* (Washington, DC: The National Academies Press, 2009).

Additionally, the order of the content standards within a grade does not dictate an order in which the indicated material is to be taught.

The CCSS are not a checklist of separate "to do" items. A single learning experience involving numbers might help students make progress toward several standards at once. For example, in kindergarten, as children begin connecting counting to cardinality (K.CC.B.4), they are challenged to develop strategies to determine whether a group of objects is greater than, equal to, or less than another group of objects (K.CC.C.6); furthermore, they can observe concrete arrangements of objects made, in order to determine how many items are in both groups (K.OA.A.2). The content standards are ***end-of-grade expectations***. Additionally, the order of the content standards within a grade does not dictate an order in which the indicated material is to be taught. For example, content standard 1.OA.A.1 sets an expectation that grade 1 students will use addition and subtraction within 20 to solve word problems involving various problem-type situations. This standard is the first one listed, but it is by no means the first standard a student would typically meet during the year. This is a grade 1 example, but the point is relevant in every grade, including high school. Standards and curriculum are two different things. The curriculum should enable all students to meet the standards in a supportive environment that emphasizes grade-appropriate understanding, including explanation of concepts and fluency with key grade-level mathematical operations such as counting, addition, and subtraction.

Kindergarten establishes foundations for counting and cardinality that lead to operations and algebraic thinking. Students begin the process of building the mathematical habits of mind that are described in the CCSS Standards for Mathematical Practice. With proper understanding of the mathematical foundations that are taught in kindergarten, students are more likely to be successful later in mathematics.[4] To give young children the time they need to learn these fundamentals in an age-appropriate environment of repeated experiencing and sense-making, kindergarten focuses strongly on number (see the following "Content Emphases by Cluster" section).[5] It is important to remember that kindergarten is a place where educators can close the gap between students who enter kindergarten with little number knowledge and those who enter with much more.

Focus and Coherence in Grades K–2[6]

For years, national reports have called for greater focus (i.e., narrowing the scope of content in each grade and deepening the time and energy spent) on fewer topics in U.S. mathematics education. The

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[4] See Geary DC, Hoard MK, Nugent L, Bailey DH. Adolescents' Functional Numeracy Is Predicted by Their School Entry Number System Knowledge. *PLoS ONE* 8(1) (2013); Jordan, Nancy C.; Kaplan, David; Ramineni, Chaitanya; Locuniak, Maria N. Early Math Matters: Kindergarten Number Competence and Later Mathematics Outcomes, *Developmental Psychology*, Vol 45(3), May 2009, 850-867 (2009).

[5] For more information, see the *K–5 Progressions Document for Counting and Cardinality and Operations and Algebraic Thinking*, available at http://math.arizona.edu/~ime/progressions/ (2014).

[6] Parts of this section are excerpted and adapted from the *K–8 Publishers' Criteria for the Common Core State Standards for Mathematics*, available at http://www.corestandards.org/assets/Math_Publishers_Criteria_K-8_Summer%202012_FINAL.pdf. (2014)

Trends in International Mathematics and Science Study (TIMSS)[7] and other international studies, which compare education systems in terms of their organization, curricula, and instructional practices in relation to their corresponding student achievement, have concluded that mathematics education in the United States is a "mile wide and an inch deep." Such an unfocused curriculum translates to less time per topic. Less time means less depth and moving on in the curriculum without many students having understood what has been taught. By design, some topics that are familiar to primary-grades teachers are absent from the K–2 standards. Implementing the standards requires moving some topics traditionally taught in earlier grades up to higher grades—sometimes to much higher grades. Teaching fewer topics, so that students can learn those topics more deeply, supports the promise that states have made to their students by adopting the CCSS: greater achievement at the college- and career-ready level, greater depth of understanding of mathematics, and a rich classroom environment in which reasoning, sense-making, applications, and a range of mathematical practices all thrive.

The strong focus of the CCSS in early grades is developing an understanding of whole number and of whole-number operations. Algebra is embedded throughout grades K–2 as students solve problems with unknowns in various positions and develop mental strategies for number operations. Concepts of measurement deepen children's understandings of number in the early grades and lay foundations for work in later grades. Additional and supporting clusters, such as Geometry, can provide different access points to, and deeper understanding of, those areas that have a strong focus in the early-grade standards.

Very early work with numbers is rich and intricate. For example, there is much more to counting than initially meets the eye. It takes time for a student to transition from rote counting (merely reciting the number words) to cardinal counting (telling how many). In order for this transition to happen, students might not reliably pair numbers 1:1 with objects (K.CC.B.4a) and might not realize that the last number said indicates the total number of objects (K.CC.B.4b). Another aspect of cardinal counting is to understand that each successive number name refers to a quantity that is one larger (K.CC.B.4c). For example, if a student knows there are five marbles in her hand, and another marble is placed in her hand, does the student find the new total by counting all of the marbles again, starting from 1, or does she simply say "6"? (Note that the latter cannot happen unless a student also knows how to rote-count forward from a number other than 1 [K.CC.A.2].) Finally, cardinal counting is a mental and physical performance that takes practice. Students may use techniques that lead to more accurate counting, such as counting objects from left to right, setting aside already-counted objects, saying (for example) "First I'll do the ducks, then the geese," and so on.

> ***The strong focus of the CCSS in early grades is developing an understanding of whole number and whole-number operations.***

In arithmetic, intricate, difficult, and necessary things form prerequisites for other intricate, difficult, and necessary things. Such aspects of mathematics as the linguistic patterns (and the deviations from those patterns) in the count sequence; the place-value system, with its three linked notions (concrete, representational, abstract) of base-ten units, recursive composing and decomposing (bundling and

[7] Mullis, I. V. S., Martin, M. O., Foy, P., and Arora, A. *TIMSS 2011 International Results in Mathematics* (Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College, 2012).

unbundling), and positional notation; and the 12 basic kinds of addition and subtraction situations (and that is just the one-step problems) are difficult and necessary topics that are essential for success in later mathematics. Each of these things is a world in itself, and the connections between these areas are important and numerous.

Without the foundations of number in place, students are unlikely to succeed in later mathematics. Research shows that early number sense predicts later functional literacy, which in turn matters for students' future life outcomes. Research also shows how early number sense predicts later math scores for all content strands. Educators' focus on the major work of each grade, as detailed in this K–2 Model Content Frameworks document, helps to keep students on track to college and career readiness. Therefore, in order to give young children the time they need to learn these fundamentals thoroughly in a grade-appropriate environment, the primary grades focus strongly on the major work of the grade, detailed further in the following individual grade-level sections of the K–2 Model Content Frameworks.

Guidance Regarding the Use of Resources in Mathematics

Even after years of implementation, considering the degree to which existing materials align to the standards continues to be important. This is often done via crosswalking exercises. Such exercises are sometimes approached simplistically, as a process of topic matching. However, it is critical to note that individual content standards are carefully crafted statements; they do not simply name topics. Therefore, coverage of topics is not a guarantee of alignment, and coverage may even affect alignment negatively when that coverage is wide and/or shallow. Cluster headings often unify the standards in a cluster by communicating the standards' joint intent. Aligning to the standards requires taking into account the guidance to be gained from cluster headings, grade-level introductions, indicators of opportunities for modeling or use of an applied approach, and so forth. In the context of a multi-grade progression, alignment also means treating the content in ways that take into account the previous stage of the progression and anticipate the next stage.

One purpose of the K–2 Model Content Frameworks for mathematics is to provide educators with guidance on the implementation of the CCSS, particularly with respect to the needs of states and districts as they develop, obtain, or revise materials to meet the standards. Therefore, this section suggests a number of important criteria for reviewing existing resources or for the development of additional curricular or instructional materials if needed. These criteria are presented in the form of the following list, which could support "strongly agree" to "strongly disagree" responses in any given case:

- Materials help students meet the indicated Standards for Mathematical Content.
- Materials also equip teachers and students to develop the varieties of expertise described in the Standards for Mathematical Practice.
- Materials are mathematically correct.
- Materials are motivating to students. The beauty and applied power of the subject is evident.
- Materials are engaging for a diverse body of students. This engagement exists side by side with the practice and hard thinking that is often necessary for learning mathematics.
- Materials reflect the standards by connecting content and practices while demanding conceptual understanding, procedural skill and fluency, and application. Specific aspects of achieving this balance include:
    - *Balance of tasks and activities.* Activities, tasks, and problems for students exhibit balance

along various dimensions. For example, some activities and tasks target procedural skill and fluency alone; others target conceptual understanding; others target application; and still others target skill, understanding, and application in equal measure. Some exercises are brief practice exercises; others require longer chains of reasoning. Some are abstract; others are contextual. Well-chosen tasks demonstrate the importance of mathematics in daily living for students, including connecting to other areas of students' interest, such as population growth and history, data and sports, and financial decision-making.

  - *Balance in how time is spent.* Materials allow time for whole-class or group discussion and debate, time for solitary problem solving and reflection, and time for thoughtful practice and routine skill building. Individual problem solving and explanation of mathematical thinking may be intertwined several times during a class.
  - *Common sense in achieving balance:* Not every task, activity, or workweek has to be balanced in the aforementioned ways. It is reasonable to have phases of narrow intensity, during which tasks, activities, and time are concentrated in a single mode.
- Materials explicitly draw the teacher's attention to nuances in the content being addressed and to specific opportunities for teachers to foster mathematical practices in the study of that content.
- Materials give teachers workable strategies for helping students who have special needs, such as students with disabilities, English language learners, and gifted students.
- Materials give teachers strategies for involving students in reading, writing, speaking, and listening as necessary to meet the mathematics standards—for example, to understand the meanings of specialized vocabulary, symbols, units, and expressions to support students in attending to precision (MP.6) or to engage in mathematical discourse using both informal language and precise language to convey ideas, communicate solutions, and support arguments (MP.3).

It is important to note that "coverage" is not included in the preceding list. Materials that are excellent but narrow in scope still have value; they can be combined with other like resources and supplemented as necessary. This approach is preferable to settling for a single mediocre resource that claims to cover all content.

Additional Resources

Members of the working group and the writing team for the CCSS for Mathematics have developed some resources to inform the development of curriculum and instruction aligned to the CCSS.

PARCC Resources

In addition to communications materials, the PARCC website, http://www.parcconline.org, includes several important resources, chiefly the prototypes for the K–2 formative instructional tasks, which provide both concrete real-life examples of the learning described in the K–2 Model Content Frameworks and early examples of the formative instructional tasks. Over time, this website will contain additional information that is relevant to K–2 educators, including evidence statements and additional released tasks.

Achieve, Inc., Resources

Achieve, Inc., is a nonprofit organization supporting states across a variety of policy initiatives, including implementation of college and career readiness standards. In particular, its website offers specific tools to support analysis of instructional materials for alignment to the CCSS, available at http://www.achieve.org/achieving-common-core.

Progressions

Progressions are being developed by members of the CCSS working group and writing team through the University of Arizona's Institute for Mathematics and Education. Progressions are narratives of the standards that describe how student skill and understanding in a particular domain develop from grade to grade. One of the primary uses of the progressions is to give educators and curriculum developers information that can help them develop materials for instruction aligned to the standards. The progressions documents are available at http://ime.math.arizona.edu/progressions/.

Illustrative Mathematics

Under the guidance of members of the CCSS working group as well as other national experts in mathematics and mathematics education, the Illustrative Mathematics project (http://www.illustrativemathematics.org/) will illustrate the range and types of mathematical work that students will experience in a faithful implementation of the CCSS and will publish other tools that support implementation of the standards.

Common Core Tools

Additional tools are posted on http://commoncoretools.wordpress.com, a blog moderated by Dr. William McCallum, distinguished professor and head of mathematics at the University of Arizona and mathematics lead for the CCSS for Mathematics.

Achieve the Core

Achieve the Core (http://www.achievethecore.org/) is a website, developed by members of the CCSS writing team, that provides instructional resources, including professional development modules and overviews of the key shifts in mathematics.

GRADE-BY-GRADE STANDARDS ANALYSES INTRODUCTION

The following pages provide insights into the standards for grades K–2. Readers are advised to use the CCSS for Mathematics in conjunction with this document. The K–2 Model Content Frameworks sometimes paraphrase standards, and in some cases refer to them by code only; readers will need to refer to the standards documents for the exact language of the standards.

Description of Components

For each grade, analysis is provided in several categories. The words *examples* and *opportunities* in the following categories emphasize that the analysis provided in each category is not exhaustive. For example, there are many opportunities to connect mathematical content and practices in every grade, many opportunities for in-depth focus in every grade, and so on. A comprehensive description of these features of the standards would be hundreds of pages long. ***The analyses provided in this document should be thought of as valuable starting points.***

Examples of Key Advances from the Previous Grade

This section highlights some of the major grade-to-grade steps in the progression of increasing knowledge and skill detailed in the standards. Each key advance in mathematical content also corresponds to a widening scope of problems that students can solve. Examples of key advances are highlighted to stress the need to treat topics in ways that take into account where students have been in previous grades and where they will be going in subsequent grades.

Fluency[8] Expectations or Examples of Culminating Standards

This section highlights individual standards that set expectations for fluency or that represent culminating masteries. Fluency standards are highlighted to stress the need to provide sufficient supports and opportunities for practice to help students meet these expectations. Culminating standards are highlighted to help give a sense of where important progressions are headed.

Fluency is not meant to come at the expense of understanding but is an outcome of a progression of learning and sufficient thoughtful practice. It is important to provide the conceptual building blocks that develop understanding in tandem with skill along the way to fluency; the roots of this conceptual understanding often extend to one or more grades earlier in the standards than the grade when fluency is finally expected.

Examples of Major Within-Grade Dependencies

This section highlights cases in which a body of content ***within a given grade*** depends, conceptually or logically, upon another body of content within that same grade. Examples of within-grade dependencies are highlighted to stress the need to organize material coherently within the grade. (Because of space limitations, only examples of large-scale dependencies are described in this section, but coherence is

[8] Wherever the word "fluently" appears in a content standard, it is used to mean "quickly and accurately." A key aspect of fluency in this sense is that it does not happen all at once in a single grade, but requires attention to student understanding as they progress towards college/career readiness. It is important to ensure that sufficient practice and extra support are provided at each grade, to allow all students to meet the standards that call explicitly for fluency.

important for dependencies that exist at finer grain sizes as well.)

Examples of Opportunities for Connections among Standards, Clusters, or Domains

This section highlights opportunities for connecting content in assessments, as well as in curriculum and instruction. Examples of connections are highlighted to stress the need to avoid approaching the standards as merely a checklist.

Examples of Opportunities for In-Depth Focus

This section highlights some individual standards that play an important role in the content at each grade. The indicated mathematics might be given an especially in-depth treatment, as measured, for example, by the type of assessment items; the number of days; the quality of classroom activities to support varied methods, reasoning, and explanation; the amount of student practice; and the rigor of expectations for depth of understanding or mastery of skills.[9]

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices

This section provides some examples of how students may engage in the mathematical practices as they learn the mathematics of the grade.[10] These examples are provided to stress the need to connect content and practices, as required by the standards. In addition to the concrete examples provided in each grade, the following are some general comments about connecting content and practices:

- Connecting content and practices happens in the context of ***working on problems.*** The very first Standard for Mathematical Practice (MP.1) involves making sense of problems and persevering in solving them.

- As the preceding point suggests, the Standards for Mathematical Practice interact and overlap with each other, and several may be used together in solving a given problem. ***The list of Standards for Mathematical Practice is not a checklist.***

Content Emphases by Cluster

This section describes content emphases in the standards at the cluster level for each grade. These emphases are provided because curriculum, instruction, and assessment at each grade must reflect the focus and emphasis of the standards.

Not all of the content in a given grade is emphasized equally in the standards. The list of content standards for each grade is not a flat, one-dimensional checklist; this is by design. There are sometimes strong differences of emphasis even within a single domain. Some clusters require greater emphasis than others, based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice. Without such focus, attention to the practices would be difficult and unrealistic, as would best practices such as formative assessment.

Therefore, in order to make relative emphases in the standards more transparent and useful, the K–2

[9] Note, however, that a standard can be individually important even though the indicated mathematics may require relatively little teaching time.
[10] See the *Progressions* documents for additional examples, http://ime.math.arizona.edu/progressions/ .

Model Content Frameworks clusters as ***Major***, ***Additional***, and ***Supporting*** for each grade. Some clusters that are not themselves major emphases are designed to *support* and strengthen areas of major emphasis, while other clusters that may not connect tightly or explicitly to the major work of the grade would fairly be called *additional*.

To say that some topics have greater emphasis is not to say that any content in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a mathematical education and will therefore be eligible for inclusion on PARCC assessments. The assessments will mirror the message that is communicated in this document: Major Clusters will be assessed on a majority of the assessment, Supporting Clusters will be assessed through their success at supporting the Major Clusters, and Additional Clusters will be assessed as well. The assessments will strongly focus where the standards strongly focus.

In addition to identifying the Major, Additional, and Supporting Clusters for each grade, suggestions are given in each grade for ways to connect the Supporting Clusters to the Major Clusters of the grade. Thus, rather than suggesting, even inadvertently, that some material should not be taught, this document provides direct advice for teaching material in ways that foster greater focus and coherence.

Finally, the following are some recommendations for using the cluster-level emphases:

Do:

- Use the guidance to inform instructional decisions regarding time and other resources spent on clusters of varying degrees of emphasis.
- Allow the focus on the major work of the grade to open up the time and space to bring the Standards for Mathematical Practice to life in mathematics instruction through sense-making, reasoning, arguing and critiquing, modeling, etc.
- Take the cluster-level emphases into account when evaluating instructional materials. The major work of the grade must be presented with the highest possible quality; the supporting work of the grade should support the major focus, not detract from it.
- Take the cluster-level emphasis into account when setting priorities for other implementation efforts, such as staff development, new curriculum development, or revision of existing formative or summative testing at the state, district, or school level.

Don't:

- Neglect any material in the standards. (Instead, use the information provided to connect Supporting and Additional Clusters to the other work of the grade.)
- Sort clusters from Major to Supporting, and then teach them in that order. To do so would strip out the coherence of the mathematical ideas and remove the opportunity to enhance the major work of the grade with the Supporting Clusters.
- Use the cluster headings as a replacement for the standards. All features of the standards matter—from the practices to the surrounding text to the particular wording of individual content standards. Guidance is given at the cluster level as a way to talk about the content with the necessary specificity and without going so far into detail as to compromise the coherence of the standards.

Examples of Key Advances in Kindergarten

- Students orally count to 100, beginning from any given number within 100, to support their later ability to count higher, as well as to develop a pattern of tens as they become skilled with naming the next ten (e.g. “forty-nine, fifty”).
- Students pair objects 1:1 with counting words, and they learn that the last number word tells the total number of objects in a collection (up to 20). This is called “cardinal counting,” as opposed to “rote counting” (reciting the counting words in order).
- Students use their ability to subitize (recognize small quantities at a glance) to help them compose and decompose numbers. For example, when students are using objects to show the decompositions 5 = 2 + 3 or 5 = 4 + 1, it is helpful for them to be able to subitize two or three objects.
- Students anchor to 5, realizing that 6 is one more than 5 and 4 is one less.
- Students build the crucial basis for place-value understanding of teen numbers by learning to anchor to 10 and to compose or decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8). This is also a crucial prerequisite for the grade 1 adding-and-subtracting strategy of making a ten and for the meaningful learning of writing numbers from 1 to 20.
- Students compare the number of objects in one group versus the number of objects in another group to find which has more or less, and eventually compare written numerals 1–10 to find which number describes more or less than another number.
- Students understand addition as joining collections and adding to collections, and they understand subtraction as taking collections apart or taking from collections. They represent these operations in a variety of ways.

Fluency Expectations or Examples of Culminating Standards

K.CC.A.1 Count to 100 by tens and ones.

K.CC.A.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).

It is recommended that, throughout the year, students work toward fluency in writing the numerals 0–10. Note that learning to write numerals is generally more difficult than learning to read them. It is common for students to reverse numbers at this stage (e.g., writing Ɛ for 3).[11]

K.CC.B.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

It is recommended that students become fluent in cardinal counting with sets ≤ 5 ***early in the year*** and with increasingly large sets as the year progresses.

K.CC.C.7 Compare two numbers between 1 and 10 presented as written numerals.

If students are less than fluent in number comparisons by the end of kindergarten, then they may not have mastered early number concepts. Note that K.CC.C.6 (Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies) is a precursor to K.CC.C.7 and portrays the kind of concrete work that students should be doing en route to mastering numeral-based comparisons.

K.OA.A.5 Fluently add and subtract within 5.

Given an oral or written expression with any two numbers 0–5 with a sum less than or equal to 5 (e.g., "three and one" or 3 + 1), students can find the sum reasonably quickly, and say or write it. For subtractions involving numbers of the same sizes, and given an oral or written expression (e.g., "four, take away one" or 4 – 1), students can find the difference reasonably quickly and say or write it. Some students may still need to use fingers or make drawings. Students grow in fluency throughout the year as they work with addition and subtraction situations.

Examples of Major Within-Grade Dependencies

- Much of the learning in kindergarten—K.CC.C.6, all of K.OA and K.NBT, and K.MD.B.3—depends on the foundational ability to count to answer "how many?" (K.CC.B.5), which itself is grounded in K.CC.B.4.

Examples of Opportunities for Connections among Standards, Clusters, or Domains

- In addition to laying the groundwork for place value in grade 1, working with numbers 11–19 (K.NBT.A.1) provides opportunities for cardinal counting beyond 10 (see K.CC.B.5) and for writing two-digit numbers (see K.CC.A.3). Ten frames, strips with ten ones and some loose ones, and math drawings can be helpful for this work.

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[11] Material adapted from National Research Council. *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity* (Washington, DC: The National Academies Press, 2009), p. 138.

- K.MD.B.3 provides opportunities for cardinal counting (see K.CC.B.5) and for comparing numbers (see K.CC.C.6). K.MD.B.3 also offers a context in which to decompose 10 in more than one way (see K.OA.A.3).
- K.G.A.2 and K.G.B.4 offer some opportunities for counting and comparing numbers.

Examples of Opportunities for In-Depth Focus

K.CC.B.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

Cardinal counting is itself a needed focus and is a main component of other work in the kindergarten classroom. Opportunities to develop students' understanding of cardinality abound, both within the instructional time devoted specifically to mathematics and elsewhere in the instructional day—for example, "How many plants did the class plant for the science project? What if we planted one more?"

K.OA.A.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

Through representing and solving addition and subtraction problems, students understand addition as joining and adding to and understand subtraction as separating and taking from. Initially, the meaning of addition is separate from the meaning of subtraction, and students build relationships between addition and subtraction over time, with subtraction coming to be understood as reversing the actions involved in addition and as finding an unknown addend.[12]

K.OA.A.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

The ability to decompose numbers flexibly is a key focus, and is connected with other standards, such as K.OA.A.1 and K.OA.A.2. Decomposing numbers is an important foundation for adding and subtracting numbers below ten and for beginning to understand relationships between adding and subtracting. Knowing decompositions is also one of the three prerequisites for using the grade 1 make-a-ten strategy to add numbers with a teen total (see K.OA.A.4 below).

K.OA.A.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

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[12] See pages 2 and 3 of the *K–5 Progressions Document for Counting and Cardinality and Operations and Algebraic Thinking*, available at http://math.arizona.edu/~ime/progressions/ (2014).

In grade 1, "making ten" will become a key strategy for adding and subtracting within 20; students gain the foundations for the first step of this strategy in kindergarten by finding the number that makes 10 when given another number. Over the course of the year, given frequent opportunities (e.g., a "how many fingers don't you see" game), many kindergarten children can become fluent with the pairs of numbers that make 10 and can, when a number less than 10 is named, name the "missing amount" even without looking at fingers. Showing numbers from 6 to 10 in groups of 5 (e.g., a top row of 5 circles with 1 to 5 circles below it) is a helpful visual support for finding the pairs of numbers that make ten.

K.NBT.1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Understanding each number 11–19 as a collection of ten and some ones (e.g., 16 = 10 + 6) requires extensive experience counting out and separating ten and the ones and relating these groups to the written numerals. Place-value cards in which the ones numeral can be placed over the 0 in the 10 (the ten card is twice as wide as the ones cards) can be helpful in connecting to the concepts in K.NBT.A.1, K.CC.B.5, and K.CC.A.3.

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices

Mathematical practices should be evident throughout mathematics instruction and should be centrally connected to the most important work of the grade. These mathematical practices can be summarized in the following sentence: Teachers help students to do mathematical sense-making about mathematical structure using mathematical drawings (or objects) to support mathematical explaining (MP 1 & 6; 7 & 8; 4 & 5; 2 & 3). Some brief examples of how the content of this grade might be connected to the Standards for Mathematical Practice follow.

- Kindergarten students say the number names by ones and by tens ("ten, twenty, thirty, . . .") all the way to 100 (K.CC.A.1). The structure of a number name such as "thirty-two" reflects the underlying system of place value. Attending to and using that structure (MP.7) is an important foundation for place value. See the K–5 progressions document for Counting and Cardinality and Operations and Algebraic Thinking for more information about how patterns in the number names affect learning the teen numbers (including deviations from those patterns, as in "sixteen," in which the digit in the ones place is said first but written second).
- As students count by tens (K.CC.A.1), they may make sense (MP.1) of these numbers by reciting each new number in the sequence "ten, twenty, thirty . . ." as a new child joins the children already standing in front of the classroom and showing all their fingers. The patterns in the place-value system—the structure of numbers (MP.7)—become more apparent when children say "six tens is sixty, seven tens is seventy, eight tens is eighty," etc. Children can also flash ten fingers as they count by tens to feel the ten that are added on with each count.

- When students progress from drawing realistic (artistic) pictures of situations to diagramming addition and subtraction situations using circles or other symbols, and making connections between them, they are relating the concrete to the abstract (MP.2) and making their first mathematical models (MP.4). Equations to describe these situations (such as 8 + 2 = 10) are also mathematical models.
- A student choosing to use objects, fingers, or a math drawing to represent and solve a word problem is an example of the student using an appropriate tool strategically (MP.5).

Manipulatives such as physical models of hundreds, tens, and ones, and visual models such as math drawings, are important parts of the K–2 classroom. These manipulatives and visual models should always be connected to written symbols and methods.

A note on manipulatives in grades K–2: Manipulatives such as physical models of hundreds, tens, and ones, and visual models such as math drawings, are important parts of the K–2 classroom. These manipulatives and visual models should always be connected to written symbols and methods.[13]

Content Emphases by Cluster

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than others, based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some topics have greater emphasis is not to say that any content in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a mathematical education and will therefore be eligible for inclusion on PARCC assessments. However, the assessments will strongly focus where the standards strongly focus.

The following table identifies the Major Clusters, Supporting Clusters, and Additional Clusters for kindergarten; below the table, suggestions are given for ways to connect the Supporting Clusters to the Major Clusters of the grade. Thus, rather than suggesting, even inadvertently, that some material should not be taught, this document provides direct advice for teaching grade-level material in ways that foster greater focus and coherence.

[13] See page 19 of *K–8 Publishers' Criteria for the Common Core State Standards for Mathematics*, available at http://www.corestandards.org/assets/Math_Publishers_Criteria_K-8_Summer%202012_FINAL.pdf. (2014)

Key: ■ Major Clusters; ◻ Supporting Clusters; ○ Additional Clusters

Counting and Cardinality

■ Know number names and the count sequence.

■ Count to tell the number of objects.

■ Compare numbers.

Operations and Algebraic Thinking

■ Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Number and Operations in Base Ten

■ Work with numbers 11–19 to gain foundations for place value.

Measurement and Data

○ Describe and compare measurable attributes.

◻ Classify objects and count the number of objects in categories.

Geometry

○ Identify and describe shapes.

◻ Analyze, compare, create, and compose shapes.

Examples of Linking Supporting Clusters to the Major Work of the Grade

So much is brand new to children in kindergarten that, as much as possible, everything throughout the school day should support everything else, as, for example, when language supports number.

- Even within mathematics itself, understanding, for example, that 18 is ten ones and eight more ones (K.NBT.A.1) requires, but also supports, understanding what it means to combine 10 and 8 or to take apart 18 (K.OA).
- K.MD.B.3 offers a context in which to decompose numbers less than or equal to 10 in more than one way (see K.OA.A.3).

Examples of Key Advances from Kindergarten to Grade 1

- Students gradually come to employ mental strategies (such as counting on and making ten) that make use of embedded concepts of number and the properties of addition and subtraction; by contrast, kindergarten students determine sums and differences primarily by representing problems with objects or drawings.
- Students read and write numbers through 120 and learn the early elements of place value, in particular being able to think of a ten as a unit and understanding that the digits of a two-digit number represent the number of tens in that number and the number of remaining ones.
- Students use their understanding of place value and the properties of operations to represent, explain, and perform addition and subtraction of two-digit numbers in specified cases.
- Students represent and solve a large variety of addition and subtraction problems—that is, word problems, and problems set in classroom discussions, that involve addition and subtraction situations such as adding to, taking from, putting together, taking apart, comparing, etc., with different unknown quantities in the problem.[14]
- Students write equations for a variety of reasons, such as expressing a decomposition of a number (16 = 9 + 7), expressing a piece of reasoning about numbers (9 + 7 = 9 + 1 + 6, along the way to making ten), or representing a word problem with an unknown (9 + ? = 16). Students use the equal sign appropriately, evaluate the truth of an equation, and determine unknown numbers that will make an equation true. Students make connections among concrete objects, pictorial representations, and equations.

Fluency Expectations or Examples of Culminating Standards

1.OA.C.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Fluency is demonstrated as that, given any two numbers 0–10 with a sum less than or equal to 20, students can say the sum reasonably quickly, and likewise, for related differences, given one number and a total that is 10 or less, they can reasonably quickly say the amount taken away or the unknown addend. Students grow in fluency throughout the year as they work with addition and subtraction situations.

[14] See Table 2 on page 9 of the *K–5 Progressions Document for Counting and Cardinality and Operations and Algebraic Thinking*, available at http://math.arizona.edu/~ime/progressions/ (2014). This table is based on Table 1 on page 88 of the CCSS for Mathematics.

1.OA.D.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true, and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*

This standard relates to fluency when the additions and subtractions in the equations fall within 10, as they do in the italicized examples accompanying the standard.

1.OA.D.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = ☐ – 3, 6 + 6 = ☐.*

A crucial aspect of understanding and solving such equations is knowing where the total is in addition equations (alone on one side) and in subtraction equations (before the minus sign). Also important is that students see varied equation forms, especially those with only one number on the left side of the equation.

1.NBT.C.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

Quickly finding 10 more or 10 less than a two-digit number is best thought of as an indicator of whether students have an understanding of place value for two-digit numbers.

Examples of Major Within-Grade Dependencies

- 1.OA.B.3 calls for students to "apply properties of operations" and gives the example "If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known." Similarly, knowing 13 – 3 gives a good starting place for figuring out 13 – 4. Use of properties lets students apply knowledge that they have to situations that they need to figure out.
- 1.NBT.B.2 describes the place-value foundations for 1.NBT.B.3 and 1.NBT.C.4. Comparing numbers (1.NBT.B.3) involves thinking about the sizes of tens and ones, and adding two-digit numbers (1.NBT.C.4) involves adding tens with tens and ones with ones, and sometimes composing a ten. These ideas and methods rest on an understanding of the place-value units and the use of visual models of these units in solving and explaining problems using these standards.

Examples of Opportunities for Connections among Standards, Clusters, or Domains

- A thorough understanding of how place-value language and notation represent number (cluster 1.NBT.A) is needed for meaningful calculation (cluster 1.NBT.B) in many ways—not just pencil-and-paper calculation, but mental calculation as well. For purposes of calculation, it is valuable to use the tens and ones in two-digit numbers, single-digit knowledge, and properties of the operations (1.OA.B.4). In Grade 1, calculation ranges from simple mental adding, such as 40 + 20 (add the 4 tens and 2 tens) and 58 + 6 (6 gives 2 to 58 to make 60, then 60 plus the 4 left in 6 equals 64), to the more complex cases that require composing ten ones to make a ten, such as 37 + 56.

- The study of word problems in grade 1 (1.OA.A.1, 1.OA.A.2) can be coordinated with students' growing proficiency with addition and subtraction within 20 (1.OA.C.6) and their growing proficiency with multidigit addition and subtraction (1.NBT) and can involve easier and more accurate forward methods.[15]

- Word problems can also be linked to students' growing understanding of properties of addition and the relationship between addition and subtraction. For example, put-together/take-apart problems with unknown addends can show subtraction as finding an unknown addend (see the "Problem Types" section).[16]

- Units are a connection between place value (1.NBT) and measurement (1.MD). Working with place value depends on having a sense of the sizes of the base-ten units and being able to see a larger unit as composed of smaller units within the system. As measurement develops through the grades, measurement also depends on having a sense of the sizes of units and being able to see a larger unit as composed of smaller units within the system. In later grades, unit thinking will become important throughout arithmetic, including in the development of multidigit multiplication and division algorithms and the development of fraction concepts and operations.[17]

- Measurement standards 1.MD.A.1 and 1.MD.A.2 together support and provide a context for the 1.OA.A.1 goal of solving problems that involve comparing. To meet 1.MD.A.1, students compare the lengths of two objects by means of a third object, e.g., a length of string, that allows a "copy" of the length of an immovable object to be moved to another location to compare with the length of a movable object. When students cannot find the ***exact*** difference because of the magnitude of the numbers that arise from measurement—as may occur in comparing two students' heights—they may still compare the measurements to know which is greater (1.NBT.B.3). (Grade 2 standard 2.MD.B.6 formalizes this idea on a number-line diagram.)

Measuring a hallway using students as length units (1.MD.A.2). (The students are posited as having equal heights.)

- While students are dealing with the limited precision of only whole hours and half-hours, they must distinguish the position of the hour hand and connect it to the geometry standard 1.G.A.3, partitioning circles into halves and quarters.

- Composing shapes to create a new shape (1.G.A.2) is the spatial analogue of composing numbers to create new numbers. This concept is also connected to length measurement (1.MD.A.2) since students must visualize an object that is to be measured as being built up out of equal-sized units (see also 1.G.A.3). Though assembling two congruent right triangles into a rectangle does not use the same facts or reasoning that assembling two fives into a ten uses, the

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[15] See page 13 of the *K–5 Progressions Document for Counting and Cardinality and Operations and Algebraic Thinking*, available at http://math.arizona.edu/~ime/progressions/ (2014).
[16] See page 13 of the *K–5 Progressions Document for Counting and Cardinality and Operations and Algebraic Thinking*, available at http://math.arizona.edu/~ime/progressions/ (2014).
[17] See *Units, a Unifying Idea in Measurement, Fractions, and Base Ten*, available at http://commoncoretools.me/2013/04/19/units-a-unifying-idea/ (2013).

idea of looking at how objects in some domain (numbers or shapes) can be combined to make other objects in that domain, and looking for new true statements one can make about these combinations, is a big idea that is common across mathematics.

Examples of Opportunities for In-Depth Focus

1.NBT.B.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: 10 can be thought of as a bundle of ten ones—called a "ten" (1.NBT.B.2.A); The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones (1.NBT.B.2.B); The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones) (1.NBT.B.2.C).

Grade 1 is students' first encounter with the three linked components of the place-value system: base ten units of tens and ones, composing and decomposing (bundling and unbundling) of units, and positional place-value notation. Understanding these and their connections is the foundation of the entire NBT domain.

1.NBT.C.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

Understanding place value is not a final goal on its own; the goal is to use place-value understanding and properties of operations to add and subtract (cluster 1.NBT.B). Students learn how standard place-value notation presents units of ten and units of one, and learn how to extend their single-digit methods for adding units of ones to add units of tens, including separately counting or adding the ones and counting or adding the tens in concrete models or drawings. When adding two arbitrary two-digit numbers (with a result within 100), being able to represent this addition with objects or drawings that show the base-ten structure supports students to understand when they need to compose a ten and how to record that new ten in their written method. Having a strong mental image and ability with single-digit versions of these additions and understanding how place-value notation can record these additions is the foundation for the extension of place-value reasoning and adding of separate units to grade 2 adding of tens that require composing a hundred (e.g., in 65 + 78) and adding units of hundreds (e.g., in 265 + 478). Note that students only subtract multiples of ten (e.g., 80 – 30) in grade 1 (1.NBT.C.6) because solving problems that do not require decomposing a ten (e.g., 78 – 32) for a long time before solving problems that do require decomposing a ten can lead to the common top-from-bottom error (e.g., 73 – 49 = 36).

1.OA.A.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem

There are many distinct elementary addition and subtraction situations; students in grade 1 should work extensively with all of them.[18]

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices

Mathematical practices should be evident throughout mathematics instruction and should be centrally connected to the most important work of the grade. These mathematical practices can be summarized in the following sentence: Teachers help students to do mathematical sense-making about mathematical structure using mathematical drawings (or objects) to support mathematical explaining (MP 1 & 6; 7 & 8; 4 & 5; 2 & 3). Some brief examples of how the content of this grade might be connected to the Standards for Mathematical Practice follow.

- All work with properties (1.OA.B.3) and with understanding and using place value (e.g., 1.NBT.B.2, 1.NBT.C.4) should be seen as an investigation and use of the structure of the number system and of arithmetic (MP.7). Students' explanations of the properties and reasoning that they used in these contexts are early beginnings of the construction of (brief) logical arguments (MP.3). Examples of brief but excellent arguments at this grade level could include:
    - I know that 7 – 3 equals 4 because 4 + 3 equals 7. (This shows 1.OA.B.4 being met.)
    - I knew that 8 + 8 = 20 was wrong because 10 + 10 equals 20 and 8 is less than 10.
    - I know that 8 + 7 equals 15 because I know that 8 + 8 equals 16.
- Use of MP.8 ("Look for and express regularity in repeated reasoning") is important in the work on adding two-digit numbers (1.NBT.C.4), as described in the preceding in-depth-focus section. Students will repeatedly think about the units of ten and the units of one in their concrete models or drawings and in their recorded written methods. This work also uses MP.4 (modeling with mathematics), providing an example of a common situation in which multiple practices are involved in a given activity.
- Students in grade 1 work with some sophisticated addition and subtraction situations (1.OA.A.1), such as "Lucy has 8 fewer apples than Julie. Julie has 12 apples. How many apples does Lucy have?" Making a math drawing or using objects to model the situation is very helpful for students. The equations 12 – 8 = ?, 8 + ? = 12, and ? + 8 = 12 are all mathematical models of this situation (MP.4).

Manipulatives such as physical models of hundreds, tens, and ones, and visual models such as math drawings, are important parts of the K–2 classroom. These manipulatives and visual models should always be connected to written symbols and methods.

[18] Some situation subtypes need not be mastered until grade 2. See Table 2 on page 9 of the *K–5 Progressions Document for Counting and Cardinality and Operations and Algebraic Thinking*, available at http://math.arizona.edu/~ime/progressions/ (2014).

A note on manipulatives in grades K–2: Manipulatives such as physical models of hundreds, tens, and ones, and visual models such as math drawings, are important parts of the K–2 classroom. These manipulatives and visual models should always be connected to written symbols and methods.[19]

Content Emphases by Cluster

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than others, based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

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The following table identifies the Major Clusters, Supporting Clusters, and Additional Clusters for each grade; below the table, suggestions are given for ways to connect the Supporting Clusters to the Major Clusters of the grade. Thus, rather than suggesting, even inadvertently, that some material should not be taught, this document provides direct advice for teaching grade-level material in ways that foster greater focus and coherence.

Key: ■ Major Clusters; □ Supporting Clusters; ○ Additional Clusters

Operations and Algebraic Thinking	
■	Represent and solve problems involving addition and subtraction.
■	Understand and apply properties of operations and the relationship between addition and subtraction.
■	Add and subtract within 20.
■	Work with addition and subtraction equations.
Number and Operations in Base Ten	
■	Extend the counting sequence.

[19] See page 19 of the *K–8 Publishers' Criteria for the Common Core State Standards for Mathematics*, available at http://www.corestandards.org/assets/Math_Publishers_Criteria_K-8_Summer%202012_FINAL.pdf. (2014)

- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data

- Measure lengths indirectly and by iterating length units.
- Tell and write time.
- Represent and interpret data.

Geometry

- Reason with shapes and their attributes.

Examples of Linking Supporting Clusters to the Major Work of the Grade

- When students work with organizing, representing, and interpreting data, the process includes practicing using numbers and adding and subtracting to answer questions about the data (see the part of 1.MD.C.4 after the semicolon (". . . ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another"), and see the K–5 progressions document for Measurement and Data (data part), especially Table 1 on page 4 and the discussion of categorical data on pages 5 and 6).[20]
- Telling and writing time on digital clocks (1.MD.B.3) is a context in which one can practice reading numbers (1.NBT.A.1), a kind of "application," but no more. Relating those times to ***meanings***—such as events during a day—is not part of 1.MD.B.3, but making sense of what one is doing (MP.1) and contextualizing (MP.2) are essential elements of good mathematical practice and should be part of the instructional foreground at all times.

[20] The *K–5 Progressions Document for Measurement and Data* (data part) is available at http://commoncoretools.files.wordpress.com/2011/06/ccss_progression_md_k5_2011_06_20.pdf (2011).

PARCC MODEL CONTENT FRAMEWORK FOR MATHEMATICS FOR GRADE 2

Examples of Key Advances from Grade 1 to Grade 2

- Students read and write numbers through 1,000, extending their use of place value to include units of hundreds.
- Students use their understanding of place value to add and subtract within 1,000 (e.g., 237 + 616 or 822 – 237). They can explain what they are doing as they add and subtract, and record their written method, using visual models to support calculating and explaining. They become fluent in addition and subtraction within 100.
- For word problems, students extend their ability by solving two-step problems using addition, subtraction, or both operations. They also master more advanced one-step addition and subtraction problems in this grade (such as take from with start unknown).[21]
- Students use standard units of measure and appropriate measurement tools. They understand basic properties of linear measurement (e.g., length, distance), such as the fact that the smaller the unit, the more iterations will be needed to cover a given length.

Fluency Expectations or Examples of Culminating Standards

2.OA.B.2 Fluently add and subtract within 20 using mental strategies. By end of grade 2, know from memory all sums of two one-digit numbers.

2.NBT.B.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

Critical area 2 within the grade 2 CCSS for Mathematics introduction says "They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations." As a result, students also use efficient, accurate, and generalizable methods for fluency within 100.

2.NBT.A.2 Count within 1000; skip-count by 5s, 10s, and 100s.

2.NBT.A.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

Fluency with these relationships is important for adding and subtracting within 1000.

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[21] See Table 2 on page 9 of the *K–5 Progressions Document for Counting and Cardinality and Operations and Algebraic Thinking*, available at http://math.arizona.edu/~ime/progressions/ (2014). This table is based on Table 1 on page 88 of the CCSS for Mathematics.

2.NBT.B.8 Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.

2.MD.A.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Students require sufficient practice to measure accurately and reasonably quickly.

Examples of Major Within-Grade Dependencies

- "Understand place value" (cluster 2.NBT.A) is the foundation for "Use place value understanding and the properties of operations to add and subtract" (cluster 2.NBT.B). (Mastery of the two clusters can grow over time, in tandem with each other.) Adding and subtracting within 1,000 (2.NBT.B.7) involves adding or subtracting hundreds with hundreds, tens with tens, and ones with ones, sometimes composing or decomposing tens or hundreds. These ideas and methods rest on an understanding of the place-value units (2.NBT.A.1, building on 1.NBT.A.2) and understanding these units deepens students' understanding of place value.
- Knowing single-digit sums from memory (2.OA.B.2) is the basis for adding and subtracting multidigit numbers fluently and efficiently in general (cluster 2.NBT.B).

Examples of Opportunities for Connections among Standards, Clusters, or Domains

- Problems involving dollars, dimes, and pennies (2.MD.C.8) should be connected with the place-value learning of hundreds, tens, and ones (2.NBT.A.1), though the notation is different. A dollar is 100 cents, or a "bundle" of 10 dimes, each of which is a "bundle" of 10 pennies. Work with dollars, dimes, and pennies (without the notation) can support methods of whole-number addition (e.g., dimes are added to dimes). Addition that is appropriate with whole numbers can be explored in the new notation of money contexts (though fluency with that notation is ***not*** a standard at this grade).
- Students' work with addition and subtraction word problems (2.OA.A.1) can be coordinated with their growing skill in multidigit addition and subtraction (2.OA.B.2; cluster 2.NBT.B).
- Work with nickels (2.MD.C.8) and with telling time to the nearest five minutes on analog clocks (2.MD.C.7) should be taken together with counting by 5s (2.NBT.A.2) as contexts for gaining familiarity with repeating groups of 5 (2.OA.C.4). Recognizing time by seeing the minute hand at 3 and ***knowing*** that that signifies 15 minutes; recognizing three nickels as 15 cents; and seeing the 15-ness of a 3-by-5 rectangular array held in any position (including with neither base horizontal) will prepare for understanding, in grade 3, what the new operation of multiplication means.
- A number line (2.MD.B.6) connects numbers, lengths, and units. Number lines are first used in grade 2. A number line shows units of length; the numbers at the end points of the lengths tell how many lengths so far. Bar-graph scales (2.MD.D.10) and rulers (2.MD.A.1, 2, 3, 4) are number lines. Length units can be added and subtracted using rulers or number-line diagrams (2.MD.B.5, 6); adding lengths is an extension of adding and subtracting numbers of things, which has been a

focus in kindergarten and grade 1 and will be a focus in grade 2 OA and NBT standards. The purpose of number lines is to represent numbers, sums, and differences as lengths, rather than using lengths to solve all addition and subtraction problems.

Examples of Opportunities for In-Depth Focus

2.OA.A.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem

Using situations (from word problems, from classroom events or student experiences, and from discovered mathematical patterns) as a source of problems can help students make sense of and contextualize the operations they are learning. Students continue to relate the different basic situation types in addition and subtraction.[22] Using equations and drawn models to represent situations can facilitate understanding, explaining, and such relating.

2.NBT.B.7 Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

It takes substantial time for students to extend addition and subtraction to 1,000. Students must connect steps in the written method to what they know about place value and properties of operations, using visual models to support sense-making and explaining. Students need to be able to use general methods that will extend to larger numbers in grade 4, so that grade 3 students can concentrate on multiplication and division and on becoming fluent in their grade 2 addition and subtraction written methods within 1000.

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practice

Mathematical practices should be evident throughout mathematics instruction and should be centrally connected to the most important work of the grade. These mathematical practices can be summarized in the following sentence: Teachers help students to do mathematical sense-making about mathematical structure using mathematical drawings (or objects) to support mathematical explaining (MP 1 & 6; 7 & 8; 4 & 5; 2 & 3). Some brief examples of how the content of this grade might be connected to the Standards for Mathematical Practice follow.

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[22] See Table 2 on page 9 of the *K–5 Progressions Document for Counting and Cardinality and Operations and Algebraic Thinking*, available at http://math.arizona.edu/~ime/progressions/ (2014); as well as *Representing and Solving Addition and Subtraction Problems Mini-Assessment*, available at http://www.achievethecore.org/page/258/representing-and-solving-addition-and-subtraction-problems (2014).

- Students use MP.7 ("Look for and make use of structure") as they compose ones and tens to make tens and hundreds, and decompose hundreds and tens when they need more tens and ones. They use MP.8 ("Look for and express regularity in repeated reasoning") both as they compose and decompose such units and as they initially extend their reasoning and written methods from adding and subtracting within 100 to adding and subtracting within 1000.
- Grade 2 students use objects that remain appropriate tools (MP.5) for a lifetime: rulers, clocks, coins, and the number line (essentially an abstract ruler or measurement scale). MP.5 is about not just the ability to use tools, but the ability to choose the appropriate tool for a task. At this grade level, because students are just beginning to use a variety of tools, their utility may seem both obvious and fixed to a task: for example, a ruler measures length. For some students, for example, coming to understand the significance of counting by 5—the usefulness of that counting sequence and the situations in which it appears—may involve a choice of which of several images (nickels, hands, telling time) is most clarifying and salient to them. Generating the abstraction—in this example, the sequence 0, 5, 10, 15, etc.—may also be aided by experiences in the various domains, including the recognition that the same sequence of numbers is common to all of them. That sequence of number names expresses the regularity (MP.8) of a calculation (counting five more) that recurs in many contexts.

Manipulatives such as physical models of hundreds, tens, and ones, and visual models such as math drawings, are important parts of the K–2 classroom. These manipulatives and visual models should always be connected to written symbols and methods.

A note on manipulatives in grades K–2: Manipulatives such as physical models of hundreds, tens, and ones, and visual models such as math drawings, are important parts of the K–2 classroom. These manipulatives and visual models should always be connected to written symbols and methods.[23]

Content Emphases by Cluster

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than others, based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some topics have greater emphasis is not to say that any content in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a

[23] See page 19 of *K–8 Publishers' Criteria for the Common Core State Standards for Mathematics*, available at http://www.corestandards.org/assets/Math_Publishers_Criteria_K-8_Summer%202012_FINAL.pdf. (2014)

mathematical education and will therefore be eligible for inclusion on PARCC assessments. However, the assessments will strongly focus where the standards strongly focus.

The following table identifies the Major Clusters, Supporting Clusters, and Additional Clusters for each grade; below the table, suggestions are given for ways to connect the Supporting Clusters to the Major Clusters of the grade. Thus, rather than suggesting, even inadvertently, that some material should not be taught, this document provides direct advice for teaching grade-level material in ways that foster greater focus and coherence.

Key: ■ Major Clusters; □ Supporting Clusters; ○ Additional Clusters

Operations and Algebraic Thinking	
■	Represent and solve problems involving addition and subtraction.
■	Add and subtract within 20.
□	Work with equal groups of objects to gain foundations for multiplication.
Number and Operations in Base Ten	
■	Understand place value.
■	Use place value understanding and properties of operations to add and subtract.
Measurement and Data	
■	Measure and estimate lengths in standard units.
■	Relate addition and subtraction to length.
□	Work with time and money.
□	Represent and interpret data.
Geometry	
○	Reason with shapes and their attributes.

Examples of Linking Supporting Clusters to the Major Work of the Grade

- When students work with time and money (cluster 2.MD.C), their work with dollars, dimes, and pennies should support their understanding and skill in place value (2.NBT). Their work with nickels, with telling time to the nearest five minutes on analog clocks, with counting by 5s (2.NBT.A.2), and with arrays of five rows and/or five columns (cluster 2.OA.C) should be related.
- In cluster 2.MD.D ("Represent and interpret data"), standard 2.MD.D.10 represents an opportunity to link to the major work of grade 2. Picture graphs and bar graphs can add variety as contexts for posing and solving addition and subtraction problems. The language in 2.MD.D.10 mentions word problems (2.OA) explicitly. See the K–5 progressions document for Measurement and Data (data part).[24]

[24] For more on the connections between data work and arithmetic in the early grades see *K–5 Progressions Document for Measurement and Data* (data part) is available at http://commoncoretools.files.wordpress.com/2011/06/ccss_progression_md_k5_2011_06_20.pdf (2011).